LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2013**

MT 1500 - ALGEBRA, ANALY. GEO., CALCULUS & TRIGONOMETRY

Date: 09/05/2013 Dept. No. Max. : 100 Marks

Time: 1:00 - 4:00

PART – A (10 x 2 = 20)

Answer ALL questions:

1. Write down the nth derivative of x^2e^{5x}.
2. Find the polar subtangent and polar subnormal of the curve $r = ae^{\theta\cot\alpha}$.
3. Write the Cartesian formula for the radius of curvature.
4. Define evolute of a curve.
5. If α, β, γ are the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$, Find the value of $\sum\alpha^2$.
6. Form the equation one of whose roots is $\sqrt{5}+\sqrt{3}$.
7. Prove that $\cosh^2 x - \sinh^2 x = 1$. .
8. Write the expansion for $\cos n\theta$.
9. Find the polar of (3, 4) with respect to $y^2 = 4ax$.
10. Define an asymptote of a hyperbola.

PART – B

Answer any FIVE questions: **(5 x8 = 40)**

11. Find the slope of the tangent with the initial line for the cardioid $r = a(1-\cos\theta)$ $at\ \theta = \frac{\pi}{2}$.
12. Find the radius of curvature at the point θ of the curve

    $x = a(\cos\theta + \theta\sin\theta),\ y = a(\sin\theta - \theta\cos\theta)$.
13. Find the (p-r) equation of $r\theta = a$.
14. Solve the equation $x^3 - 19x^2 + 114x - 216 = 0$, given that the roots are in GP.
15. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, prove that

    $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha) = r - pq$.
16. Expand $\sin^3\theta\cos^4\theta$ in terms of multiples of θ.
17. P and Q are extremities of two conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

    and S is the focus. Prove that $PQ^2 - (SP - SQ)^2 = 2b^2$.
18. Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.

PART– C

Answer Any TWO Questions: **(2 x 20 = 40)**

19. a) If $y = \left(x + \sqrt{1+x^2}\right)^m$ then prove that $(1+x^2)\,y_{n+2} + (2n+1)x\,y_{n+1} + (n^2 - m^2)\,y_n = 0$.

    b) Find the angle of intersection of the curves $r = \dfrac{a}{1+\cos\theta}$ and $r = \dfrac{b}{1-\cos\theta}$.

20. a) Find the evolutes of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

    b) Solve $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.

21. a) Calculate to two places of decimals the positive root of the equation $x^3 + 24x - 50 = 0$ by Horner's method.

    b) If $\tan(\alpha + i\beta) = x + iy$ prove that $x^2 + y^2 + 2x\cot 2\alpha = 1$.

22. a) Prove that $1 - \dfrac{1}{2}\cos\theta + \dfrac{1\cdot 3}{2\cdot 4}\cos 2\theta - \dfrac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cos 3\theta + \cdots\infty = \dfrac{\cos\dfrac{\theta}{4}}{\sqrt{2\cos\dfrac{\theta}{2}}}$.

    b) If a hyperbola be a rectangular hyperbola and its equation be $xy = c^2$, prove that the locus of the middle points of chords of constant length $2l$ is $(x^2 + y^2)(xy - c^2) = l^2 xy$.

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